University of Ibn-Khaldoun, Tiaret
Faculty of Matter Sciences

1st year master Medical Physics
Solution of the exam: Physics of ionised media
Year 2023/2024

Ex 01: (5pt)

The following relation can be used:

$$\omega_p = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}} \qquad (1.5pt)$$

$$\lambda_D = \sqrt{\frac{T_e}{m_e}}\,\omega_p^{-1} = \sqrt{\frac{\varepsilon_0 T_e}{n_e e^2}} \qquad (1.5pt)$$

For a cell of a typical plasma display:

$n_e = 1.66\ 10^{17}\,\mathrm{cm}^{-3}$ and $T_e = 1.32$ eV $(1.53\ 10^4$ K) (0.5pt)+ (0.5pt)

For a fluorescent lamp:

$n_e = 10^{10}\,\mathrm{cm}^{-3}$ and $T_e = 1$ eV $(1.16\ 10^4$ K) (0.5pt)+ (0.5pt)

Ex 02: (5pt)

Replacing the Debye-Huckel potential into the Poisson equation and calculate Laplace operator in spherical coordinates

$$\nabla^2\varphi(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial\varphi}{\partial\theta}\right) + \frac{1}{r^2 sin\theta^2}\frac{\partial^2\varphi}{\partial\phi^2} \qquad (1)\ (1pt)$$

Replacing Debye-Huckel potential $\varphi(r) = \frac{e}{4\pi\varepsilon_0}\frac{exp\left(-\frac{r}{\lambda_D}\right)}{r}$ in equation (1)

gives $\nabla^2\varphi(r) = \frac{\varphi(r)}{r_D^2}$ (3) where $r_D = \lambda_D$ (2pt)

- Equation (3) is Poisson equation $\nabla^2\varphi(r) = -\frac{\rho}{\varepsilon_0}$ therefore the charge density is expressed as $\rho = -\varepsilon_0\frac{\varphi(r)}{r_D^2} = -\frac{e}{4\pi r_D^2}\frac{exp\left(-\frac{r}{\lambda_D}\right)}{r}$ (2pt)

Ex 03: (4pt)

The drift velocity is given by the relation $\overrightarrow{v_D} = \frac{\vec{E}\wedge\vec{B}}{B^2}$ ====> $v_D = \frac{E}{B} = 10^6 m/s$

(2pt) (2pt)

Ex 04: (6pt)

The cold plasma hypothesis means that the electrons only move under the influence of the field $\boldsymbol{E} = \boldsymbol{E_0} e^{i\omega t}$, so that:

$\boldsymbol{v} = \boldsymbol{v_0} e^{i\omega t}$ (1pt)

Substituting this solution into the equation

$$m_e \frac{d\boldsymbol{v}}{dt} = -e\boldsymbol{E} - m_e \nu \boldsymbol{v}$$

gives $\boldsymbol{v} = -\frac{e\boldsymbol{E}_0}{m_e} \frac{1}{\nu + i\omega} e^{i\omega t}$ (1pt)

By definition, the current density can be written $= -e\, n\, \boldsymbol{v} = \sigma \boldsymbol{E}$, from which (0.5pt)

$$\sigma = \frac{ne^2}{m_e} \frac{1}{\nu + i\omega}$$ (0.5pt)

- To develop equation wave equations for the oscillating electric and magnetic field, we first consider Faraday's law and Ampere's law

$$\nabla \times \mathbf{E} = \frac{i\omega}{c} \mathbf{B} ,$$ (1) (0.5pt)

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \sigma \mathbf{E} - \frac{i\omega}{c} \mathbf{E} .$$ (2) (0.5pt)

Eq(2) can be written

$$\nabla \times \mathbf{B} = -\frac{i\omega}{c} \epsilon \mathbf{E}$$ (3) (0.5pt)

Where $\varepsilon = 1 + i\,\frac{4\pi\sigma}{\omega}$ define the dielectric function of the plasma

Using equation (1) in eq(3) gives,

$$\boxed{\nabla^2 \mathbf{E} - \nabla(\nabla\cdot\mathbf{E}) + \frac{\omega^2}{c^2}\epsilon\mathbf{E} = 0\,.}$$

(0.5pt) (4)

Since $\nabla\cdot\mathbf{E} = 0$ and assuming a spatial dependence of $\exp(i\mathbf{k}\cdot\mathbf{x})$ then eq (4) and

(0.5pt) (0.5pt)

gives the dispersion relation for electromagnetic waves in plasma

$$\frac{\omega^2}{c^2}\epsilon = k^2$$

(5) (0.5pt)